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Thermally driven flow in a gas centrifuge with an insulated side wall

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A thermally driven steady axisymmetric flow of gas of small diffusivity in a vertical circular cylinder rotating rapidly about its axis of symmetry is studied. The side wall is a thermal insulator and the horizontal end plates are perfect conductors. The temperature of the top end plate is kept slightly higher than that of the bottom one.

The boundary-layer method is applied to solve the linearized basic equations and the following results are obtained.

(i) The axial velocity in the inner core is fully controlled by the Ekman suction on the horizontal plates and is the same as that in the case of a perfectly conducting side wall.

(ii) The closed circulation in the side-wall Stewartson $E^{\frac{1}{3}}$ layer is strongly suppressed compared with the case of a perfectly conducting side wall.

This situation is reflected in the inner temperature field, which deviates from that in the case of a perfectly conducting side wall. The critical parameter governing the solution is found to be $(\gamma - 1) Pr\, G_0 E^{-\frac{1}{3}}/4\gamma$, where Pr is the Prandtl number, γ the ratio of specific heats, E the Ekman number and G_0 the square of the Mach number based on the peripheral speed of the cylinder.

1. Introduction and summary

The gas centrifuge is a device for separating gaseous isotope mixtures such as UF_6 by a strong centrifugal acceleration. As the gas-centrifuge method is more advantageous economically than the gaseous-diffusion method for enriching uranium, the study of the gas centrifuge is one of the most urgent in the world (Abajian & Fishman 1973).

In the rotating cylinder of the gas centrifuge, counter-current flow is set up by some pumping mechanism. Because the flow profile is crucial for determining the separative power of the centrifuge (Cohen 1951; Kanagawa & Oyama 1961; Olander 1972; Matsuda 1975), a detailed study of the flow field has been necessary. For thermally driven flow of a Boussinesq fluid in a rotating cylinder, Barcilon & Pedlosky (1967) and Homsy & Hudson (1969) did the pioneering work. They showed that the solution depends upon the thermal conditions assumed at the side wall if nonlinear convective heat transport is taken into account. In the gas centrifuge, however, the centrifugal force is so large that the Boussinesq approximation is not appropriate.

Linear analyses of thermally driven flow of a compressible gas in a cylinder with perfectly conducting walls were performed by Sakurai & Matsuda (1974, hereafter referred to as I) and Nakayama & Usui (1974). They showed that the volume of a fluid element of the gas may change appreciably owing to the basic pressure variation in the radial direction. Thus a fluid element is heated or cooled by the work done by the pressure when it moves radially. This is the most important difference between a compressible gas and a Boussinesq fluid.

Despite taking into account of the effect of compressibility, these authors did not succeed in obtaining a drastic change from the Boussinesq case because they restricted themselves to the case of perfectly conducting walls. Fluid elements exchange heat via perfectly conducting walls, and thus the above-mentioned effect of the volume change is compensated for. To obtain a clearer contrast between the cases of a compressible gas and a Boussinesq fluid, therefore, we must discard our adherence to the case of perfectly conducting walls.

We study thermally driven flow of a compressible gas in a rapidly rotating cylinder and show that the solution strongly depends upon the thermal conditions assumed at the cylinder walls even if the convective heat transport is not taken into account. This approach is desirable also from a practical point of view as the walls of modern centrifuges are made of thin metal or carbon fibres. The amount of heat transport along thin walls is negligible. Thus, if we can neglect radiative heat loss from or input to the walls, they can be considered as an insulator rather than a conductor. We want to study the case of thermally insulated walls as a typical example.

In this paper, we restrict ourselves to the case of a thermally insulated side wall and perfectly conducting end plates. The opposite case will be treated in a forthcoming paper. The temperature of the top plate is kept higher than that of the bottom. For the sake of simplicity, the difference between the top and the bottom temperature is assumed to be so small that nonlinear effects can be neglected. The viscosity and thermal conductivity of the gas are assumed to be functions of the temperature only and to be small. Although many interesting phenomena may appear when we introduce a source–sink distribution on the horizontal end plates according to the results in the case of a conducting side wall (Nakayama & Usui 1974; Matsuda, Sakurai & Takeda 1975, hereafter referred to as II; Hashimoto 1975), we do not take this effect into account.

Before discussing the mathematical treatment in detail, we summarize our results. Our assumption of small viscosity and conductivity enables us to divide our flow region into an inner core and horizontal and side-wall boundary layers. As will be discussed in the following sections, our method of analysis is based on a boundary-layer treatment of the linearized basic equations. These approximations are justified by our assumptions and the method of analysis is completely analogous to that in I and II except for minor differences in notation.

The interesting physical aspects of our results, summarized in figures 1–4, are as follows. First, the meridional flow in the inner core is axial, and is the same as that in the case of a perfectly conducting side wall. This is due to the fact that the

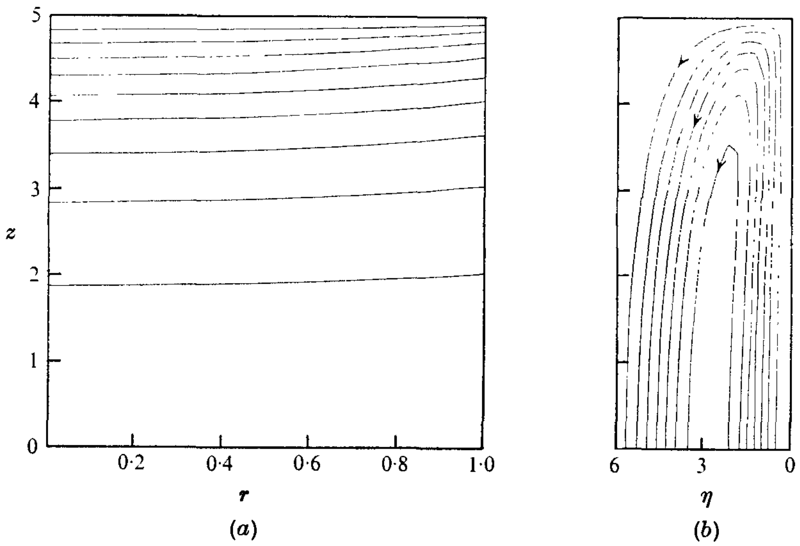

FIGURE 1. (*a*) Isotherms in inner core (T = 0–1 on a linear scale). (*b*) Streamlines of the closed circulation in the side-wall Stewartson $E^{\frac{1}{4}}$ layer (non-dimensional flux between adjacent curves is 0·005). $\alpha = hE^{-\frac{1}{4}} = 1$, aspect ratio $A = 5$.

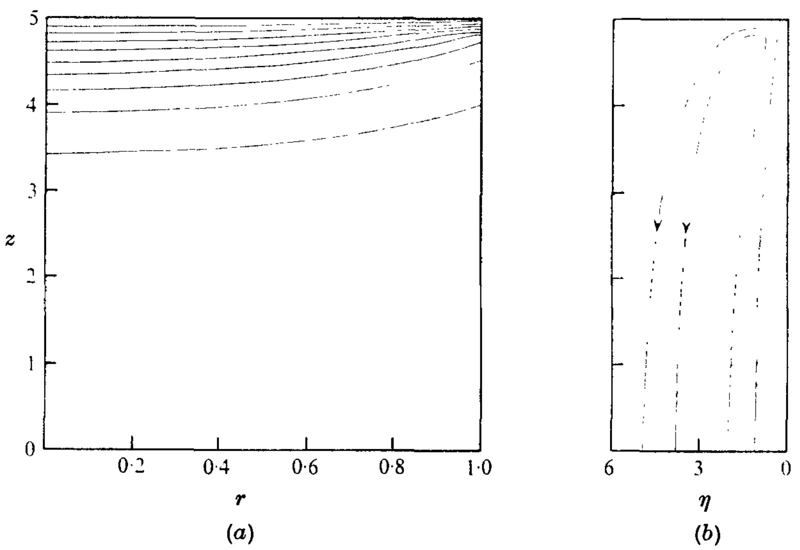

FIGURE 2. (*a*) Isotherms in inner core (T = 0–1 on a linear scale). (*b*) Streamlines of the closed circulation in the side-wall Stewartson $E^{\frac{1}{4}}$ layer (non-dimensional flux between adjacent curves is 0·005). $\alpha = 10$, $A = 5$.

axial component of the velocity is fully determined by the Ekman-layer suction on the end plates, which are still perfectly conducting in the present case. Second, the closed circulation in the side-wall Stewartson layer is suppressed. As was mentioned above, the heat produced by the radial motion of the gas in the side-wall layer can be removed through the perfectly conducting wall.

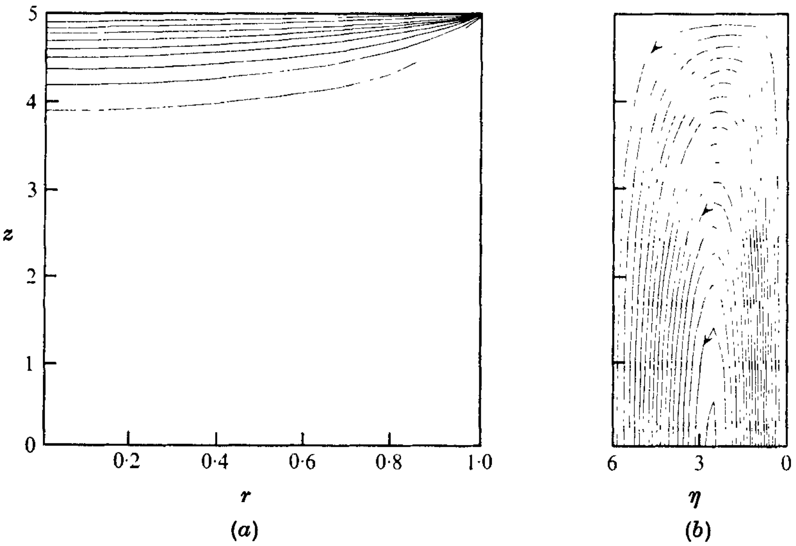

FIGURE 3. (a) Isotherms in inner core (T = 0–1 on a linear scale). (b) Streamlines of the closed circulation in the side-wall Stewartson $E^{\frac{1}{4}}$ layer (non-dimensional flux between adjacent curves is 0·005). $A = 5$. (a) $\alpha = \infty$. (b) $\alpha = 0$.

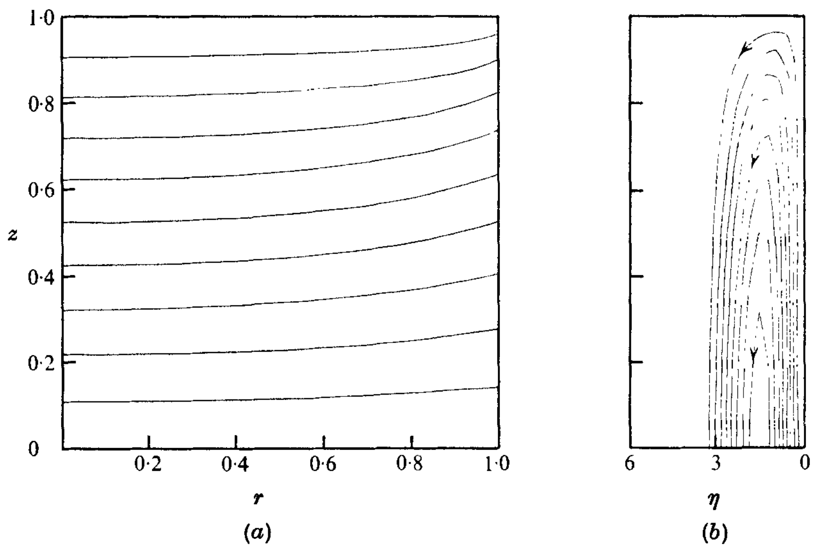

FIGURE 4. (a) Isotherms in inner core (T = 0–1 on a linear scale). (b) Streamlines of the closed circulation in the side-wall Stewartson $E^{\frac{1}{4}}$ layer (non-dimensional flux between adjacent curves is 0·005). $A = 1$, $\alpha = 1$.

Because a thermally insulated wall does not allow this process, the closed circulation is suppressed. This situation is also reflected in the temperature field in the inner core because the heat produced in the side-wall layer must be removed through the inner core. This trend may be seen by comparing figures 1 and 2. Figure 3 (a) shows the inner temperature field in the limiting case of high compressibility, while figure 3 (*b*) shows the closed circulation in the incompressible case. Therefore the present results are dramatically displayed by a comparison of figures 1 (*a*) and 3 (*a*) and figures 3 (*b*) and 2 (*b*).

In § 2, the linearized basic equations and boundary conditions are given. In § 3, we give the flow in the inner core and the horizontal Ekman layers. The flow in the side-wall boundary layer is discussed in § 4. In § 5, the equations derived in §§ 3 and 4 are solved by an expansion method. A discussion is given in § 6.

2. Formulation

Let us consider a gas in a vertical circular cylinder of radius L and height $2H$ rotating with angular velocity Ω about its axis of symmetry. The side wall of the cylinder is thermally insulated while the top and the bottom end plates are perfectly conducting and kept at fixed temperatures $\tilde{T}_0+\Delta\tilde{T}$ and $\tilde{T}_0-\Delta\tilde{T}$, respectively, where $\Delta\tilde{T}$ is small in comparison with $\tilde{T}_0$. Because the characteristic centrifugal acceleration Ω^2L is about 10^5 times as large as gravity in a typical gas centrifuge, we can neglect the effect of gravity.

In the basic state, in which $\Delta\tilde{T}$ is zero, the gas rotates rigidly with the cylinder, and the pressure $\tilde{P}_R$ and the density $\tilde{\rho}_R$ are determined by a static balance between the centrifugal force and the pressure:

$$\tilde{P}_R = \tilde{P}_2\epsilon_R, \quad \tilde{\rho}_R = M\tilde{P}_R/R\tilde{T}_0, \tag{2.1}$$

$$\epsilon_R = \exp\left[M\Omega^2(\tilde{r}^2-L^2)/2R\tilde{T}_0\right], \tag{2.2}$$

where M and R are the mean molecular weight of the gas and the universal gas constant, respectively, and $\tilde{r}$ is the radial distance from the axis of rotation; the tildes denote dimensional quantities and the suffix 2 evaluation at the periphery.

Defining the thermal Rossby number δ by

$$\delta = \Delta\tilde{T}/\tilde{T}_0, \tag{2.3}$$

we introduce the following non-dimensional variables:

$$\begin{gathered}(r,z) = (\tilde{r}/L,\, \tilde{z}/L), \quad (u,v,w) = (\tilde{q}_r, \tilde{q}_\theta, \tilde{q}_z)/L\Omega\delta, \\ T = (\tilde{T}-\tilde{T}_0)/\tilde{T}_0\delta, \quad p = (\tilde{p}-\tilde{p}_R)/\tilde{p}_R\delta,\end{gathered} \tag{2.4}$$

where $(\tilde{q}_r, \tilde{q}_\theta, \tilde{q}_z)$ are the original velocity components in the rotating frame. Note that the notation and the non-dimensionalization are slightly different from those in I and II.

Neglecting terms of order δ by our basic assumption, we get the non-dimensional linear equations

$$\operatorname{div}\mathbf{q} + G_0 ru = 0, \tag{2.5}$$

$$-2v + rT + \frac{1}{G_0}\frac{\partial p}{\partial r} = \frac{E}{\epsilon_R}\left(Lu + \frac{1}{3}\frac{\partial}{\partial r}\operatorname{div}\mathbf{q}\right), \tag{2.6}$$

$$2u = (E/\epsilon_R)Lv, \tag{2.7}$$

$$\frac{1}{G_0}\frac{\partial p}{\partial z} = \frac{E}{\epsilon_R}\left(\Delta w + \frac{1}{3}\frac{\partial}{\partial z}\operatorname{div}\mathbf{q}\right), \tag{2.8}$$

$$-\frac{(\gamma-1)\,Pr\,G_0}{\gamma} ru = \frac{E}{\epsilon_R}\Delta T, \tag{2.9}$$

where $$\operatorname{div}\mathbf{q} = \frac{1}{r}\frac{\partial}{\partial r}(ru) + \frac{\partial w}{\partial z}, \quad L = \Delta - \frac{1}{r^2}, \quad \Delta = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}, \tag{2.10}$$

$$G_0 = M(L\Omega)^2/R\tilde{T}_0, \quad E = \mu/\tilde{\rho}_2\Omega L^2, \quad \epsilon_R = \exp\left[\tfrac{1}{2}G_0(r^2-1)\right], \tag{2.11}$$

Pr is the Prandtl number, γ the ratio of specific heats and μ the coefficient of viscosity. We base our Ekman number E on the peripheral density in contrast to those in I and II, in which we used the density at the axis of the cylinder. We prefer our present definition from a practical point of view, because $\tilde{\rho}_2$ does not change much from one centrifuge to another to prevent solidification of the operating gas. The order of magnitude of E is about 10^{-6}–10^{-7}. Note that the combination E/ϵ_R, which we call the effective Ekman number or the local Ekman number, appears as an important parameter in the basic equations. This parameter ranges from 10^{-7} at the periphery to 10^{-2} at the centre. The parameter G_0, which is the square of the Mach number based on the peripheral speed, is about 20; the radius L and height $2H$ are of the order of 10 cm and 100 cm, respectively. The Prandtl number is about 1 and γ is 1·067.

If we neglect rarefied-gas effects, the gas velocity must vanish at the walls and its temperature must coincide with that of the boundary there:

$$u = v = w = 0 \quad \text{on} \quad z = \pm A \quad \text{and} \quad r = 1, \tag{2.12}$$

$$T = \pm 1 \quad \text{on} \quad z = \pm A, \qquad \partial T/\partial r = 0 \quad \text{on} \quad r = 1, \tag{2.13}$$

where $A\ (= H/L)$ is an aspect ratio.

3. The inner core and the Ekman layer

The inner core

The variables are scaled in the inner core according to

$$u = Eu_i, \quad v = v_i, \quad w = E^{\frac{1}{2}}w_i, \quad T = T_i, \quad p = G_0 p_i, \tag{3.1}$$

where quantities with a suffix i are of order unity. Inserting (3.1) into (2.5)–(2.9) and retaining the terms of lowest order with respect to E, we obtain

$$\partial w_i/\partial z = 0, \tag{3.2}$$

$$-2v_i + rT_i + \partial p_i/\partial r = 0, \tag{3.3}$$

$$2u_i = Lv_i/\epsilon_R, \quad \partial p_i/\partial z = 0, \qquad (3.4), (3.5)$$

$$-4hru_i = \Delta T_i/\epsilon_R, \tag{3.6}$$

where $h = (\gamma-1)\,Pr\,G_0/4\gamma$.

Eliminating p_i from (3.3) and (3.5) and taking into account the antisymmetry of T_i and v_i with respect to z, we get

$$v_i = \tfrac{1}{2}rT_i. \tag{3.7}$$

Elimination of u_i from (3.4) and (3.6) gives

$$\frac{\partial^2 T_i}{\partial r^2}+\frac{1}{r}\frac{1+3hr^2}{1+hr^2}\frac{\partial T_i}{\partial r}+\frac{\partial^2 T_i}{\partial z^2}=0, \tag{3.8}$$

where use has been made of (3.7).

The Ekman layer

Introducing a stretched co-ordinate $\zeta = E^{-\frac{1}{2}}(A-jz)$, where $j=1$ at the top and $j=-1$ at the bottom, we can define Ekman-layer variables (denoted by carets) as

$$\hat{u} = -jr\,e^{-\sigma\zeta}\sin(\sigma\zeta)/2(1+hr^2)^{\frac{1}{2}}, \tag{3.9}$$

$$\hat{v} = -jr\,e^{-\sigma\zeta}\cos(\sigma\zeta)/2(1+hr^2), \tag{3.10}$$

$$\hat{T} = jhr^2\,e^{-\sigma\zeta}\cos(\sigma\zeta)/(1+hr^2), \tag{3.11}$$

$$\hat{w} = \frac{1}{8\sigma(1+hr^2)^{\frac{3}{2}}}[4+hr^2+G_0r^2(1+hr^2)]\,e^{-\sigma\zeta}(\cos\sigma\zeta+\sin\sigma\zeta) - \frac{r\zeta}{2\sigma(1+hr^2)^{\frac{1}{2}}}\frac{d\sigma}{dr}e^{-\sigma\zeta}\sin\sigma\zeta, \tag{3.12}$$

where

$$\sigma = [\epsilon_R^2(1+hr^2)]^{\frac{1}{4}}. \tag{3.13}$$

The inner axial velocity w_i can be determined from (3.2) and (3.12) as

$$w_i = -\frac{1}{8\epsilon_R^{\frac{1}{2}}(1+hr^2)^{\frac{3}{4}}}\left(1+G_0r^2+\frac{3}{1+hr^2}\right). \tag{3.14}$$

The boundary conditions on (3.8) on the top and bottom end plates are

$$T_i = j(1+hr^2)^{-1} \quad \text{on} \quad z = jA. \tag{3.15}$$

4. The side-wall Stewartson $E^{\frac{1}{3}}$ layer

In the present problem, both v_i and T_i are antisymmetric functions of z, and only an $E^{\frac{1}{3}}$ Stewartson layer arises. The proper scaling for the variables in the $E^{\frac{1}{3}}$ layer is

$$u = E^{\frac{1}{3}}\bar{u}, \quad v=\bar{v}, \quad w=\bar{w}, \quad T=h\bar{T}, \quad p = G_0E^{\frac{1}{3}}\bar{p}, \quad 1-r = E^{\frac{1}{3}}\eta, \tag{4.1}$$

where quantities with an overbar are boundary-layer variables of order unity and η is a stretched radial co-ordinate. Note that T is scaled in a manner different from that in I.

Substitution of (4.1) into (2.5)–(2.9) gives

$$\partial\bar{u}/\partial\eta - \partial\bar{w}/\partial z = 0, \quad -2\bar{v}+h\bar{T}-\partial\bar{p}/\partial\eta = 0, \qquad (4.2), (4.3)$$

$$2\bar{u} = \partial^2\bar{v}/\partial\eta^2, \quad \partial\bar{p}/\partial z = \partial^2\bar{w}/\partial\eta^2, \quad -4\bar{u} = \partial^2\bar{T}/\partial\eta^2. \qquad (4.4)\text{–}(4.6)$$

Noting (4.2), which is similar in form to the continuity equation of an incompressible fluid, we can introduce a stream function $\bar{\psi}$:

$$\bar{u} = \partial\bar{\psi}/\partial z, \quad \bar{w} = \partial\bar{\psi}/\partial\eta. \tag{4.7}$$

Eliminating $\bar{u}$ from (4.4) and (4.6), and noting that all boundary-layer quantities must vanish as $\eta \to \infty$, we have the thermal-wind relation

$$\bar{v} = -\tfrac{1}{2}\bar{T}. \tag{4.8}$$

Elimination of $\bar{u}$, $\bar{v}$, $\bar{w}$ and $\bar{p}$ from (4.2)–(4.6) leads to a governing equation for $\bar{T}$:

$$\partial^6 \bar{T}/\partial \eta^6 + 4(1+h)\,\partial^2 \bar{T}/\partial z^2 = 0. \tag{4.9}$$

Once $\bar{T}$ is known, we can determine $\bar{v}$ from (4.8) and $\bar{\psi}$ from

$$\bar{\psi} = -\frac{1}{4}\int_{-A}^{z} (\partial^2 \bar{T}/\partial \eta^2)\, dz, \tag{4.10a}$$

where the integration constant is determined by the relation $\bar{\psi}(z = -A) = 0$. From (4.3), (4.5) and (4.8), we have the corresponding formula

$$\partial \bar{T}/\partial z = (1+h)^{-1}\,\partial^4 \bar{\psi}/\partial \eta^4. \tag{4.10b}$$

The boundary conditions at $r = 1$, i.e. $\eta = 0$, are

$$\bar{\psi} = \partial \bar{\psi}/\partial \eta = v_i + \bar{v} = 0, \tag{4.11}$$

$$\alpha(\partial \bar{T}/\partial \eta) = \partial T_i/\partial r, \tag{4.12}$$

where $\alpha \equiv hE^{-\frac{1}{3}}$. The parameter α is in practice of the order of 10. Let us consider it to be of order unity, however, for the sake of convenience. The boundary conditions on the side wall cannot be expressed solely in terms of boundary-layer variables, and are expressed as coupling conditions between the inner and the boundary-layer flow. The boundary conditions (4.11) are rewritten in terms of temperatures:

$$\bar{T} = T_i, \quad \partial^2 \bar{T}/\partial \eta^2 = \partial^3 \bar{T}/\partial \eta^3 = 0. \tag{4.13), (4.14}$$

Thus our problem is to solve (3.8) and (4.9) simultaneously subject to (3.15) and (4.12)–(4.14). This strong coupling between the inner and boundary-layer flow via the side-wall boundary conditions is the most important point in the present treatment. To get a clearer understanding of the situation, let us express the contents of the mathematical treatment in physical terms. The function of the side-wall Stewartson layer is to mediate between the thermal wind in the inner core and the boundary condition on the side wall. Therefore the azimuthal component of the boundary-layer flow must be of order unity. This azimuthal component induces a closed circulation of order $E^{\frac{1}{3}}$ in the boundary layer, via the balance between Coriolis and viscous forces [see (4.4)]. The radial component of this closed circulation causes changes in the volume of fluid elements and associated heating or cooling of these elements owing to the work done by the pressure. Although this heating or cooling is balanced by thermal conduction in the gas [see (4.6)], it is inevitable that an excess or a lack of heat appears on account of this boundary-layer flow. This excess or lack cannot be compensated for by the thermally insulated side wall so this must be done by the inner temperature field. Thus the closed circulation in the boundary layer strongly affects the temperature field in the inner core, and vice versa.

5. Method of solution

According to the h-expansion method proposed in I, let us take h as a small parameter and expand with respect to it:

$$\bar{T} = \tau_0 + h\tau_1 + \ldots, \quad T_i = \theta_0 + h\theta_1 + \ldots. \tag{5.1}$$

In the present work, we retain only the leading terms τ_0 and θ_0. Although h is assumed to be small in comparison with unity, α ($= hE^{-\frac{1}{3}}$) is of order unity.

Substitution of (5.1) into (4.9) and neglect of terms of higher order with respect to h gives

$$\partial^6\tau_0/\partial\eta^6 + 4\partial^2\tau_0/\partial z^2 = 0. \tag{5.2}$$

By a similar procedure, we get an equation for the inner temperature:

$$\Delta\theta_0 = 0. \tag{5.3}$$

The boundary conditions (4.12)–(4.14) and (3.15) reduce to

$$\left.\begin{aligned} \alpha(\partial\tau_0/\partial\eta) &= \partial\theta_0/\partial r \quad (5.4)\\ \tau_0 &= \theta_0 \quad (5.5)\\ \partial^2\tau_0/\partial\eta^2 &= \partial^3\tau_0/\partial\eta^3 = 0 \quad (5.6) \end{aligned}\right\} \quad \text{on} \quad r = 1 \quad (\eta = 0), \quad -A < z < A,$$

$$\theta_0 = \pm 1 \quad \text{on} \quad z = \pm A, \quad 0 \leqslant r < 1. \tag{5.7}$$

The temperature τ_0 is expanded in the cosine series

$$\tau_0 = \sum_{m=1}^{\infty} f_m(\eta)\cos[(2m-1)\pi(z+A)/2A], \tag{5.8}$$

where only odd terms are retained because of the antisymmetry of τ_0 with respect to $z = 0$. This cosine series can be differentiated term by term with respect to z (Williams 1973, p. 18). The function $\partial\tau_0/\partial z$ can also be differentiated term by term because $\partial\tau_0/\partial z$ vanishes at $z = \pm A$ [consider (4.10b) and the condition $\bar{\psi}(z = \pm A) = 0$]. Substitution of (5.8) into (5.2) subject to these conditions gives

$$d^6 f_m/d\eta^6 - \omega_m^6 f_m = 0, \tag{5.9}$$

where $\omega_m = [(2m-1)\pi/A]^{\frac{1}{3}}$. The solution of (5.9) subject to (5.6) is

$$f_m(\eta) = \tfrac{1}{2}f_m(0)\left[\exp(-\omega_m\eta) + 2\times 3^{-\frac{1}{2}}\exp(-\tfrac{1}{2}\omega_m\eta)\cos(\tfrac{1}{2}\times 3^{\frac{1}{2}}\omega_m\eta - \tfrac{1}{6}\pi)\right], \tag{5.10}$$

where $f_m(0)$ is a constant to be determined later.

The function θ_0 is divided according to

$$\theta_0 = z/A + \Phi_0(r,z), \tag{5.11}$$

where

$$\left.\begin{aligned} \Delta\Phi_0 &= 0,\\ \Phi_0 = 0 \quad &\text{at} \quad z = \pm A. \end{aligned}\right\} \tag{5.12}$$

Let us expand Φ_0 in a sine series:

$$\Phi_0 = \sum_{n=1}^{\infty} g_n(r)\sin[n\pi(z+A)/A], \tag{5.13}$$

where the antisymmetric nature of Φ_0 has been used. Because Φ_0 is zero at $z = \pm A$, this series can be differentiated term by term. As the value of $\partial\Phi_0/\partial z$ at $z = A$ is the same as that at $z = -A$ by the antisymmetry of Φ_0, the Fourier series for $\partial\Phi_0/\partial z$ can also be differentiated term by term. Thus we obtain

$$\frac{d^2 g_n}{dr^2} + \frac{1}{r}\frac{dg_n}{dr} - \beta_n^2 g_n = 0, \tag{5.14}$$

where $\beta_n = n\pi/A$. The solution of (5.14) which is regular at the axis is

$$g_n = a_n I_0(\beta_n r), \tag{5.15}$$

where I_0 is a modified Bessel function of order zero and a_n is a constant to be determined later.

By substitution of the above formulae, the boundary conditions (5.4) and (5.5) reduce to

$$-\tfrac{1}{2}\alpha \sum_{m=1}^{\infty} f_m(0)\,\omega_m \cos\frac{(2m-1)\pi(z+A)}{2A} = \sum_{n=1}^{\infty} a_n \beta_n I_1(\beta_n) \sin\frac{n\pi(z+A)}{A}, \tag{5.16}$$

$$\sum_{m=1}^{\infty} f_m(0) \cos\frac{(2m-1)\pi(z+A)}{2A} = \frac{z}{A} + \sum_{n=1}^{\infty} a_n I_0(\beta_n) \sin\frac{n\pi(z+A)}{A}. \tag{5.17}$$

Thus the problem is to solve (5.16) and (5.17) for the $f_m(0)$ and a_n. There are two methods of solution. First, the left-hand side may be rearranged as the series expansion of $\sin[n\pi(z+A)/A]$, and we get

$$-\frac{4A\alpha}{\pi} \sum_{m=1}^{\infty} \left[\frac{(2m-1)\pi}{A}\right]^{\frac{1}{2}} \frac{f_m(0)}{4n^2-(2m-1)^2} = \pi I_1\left(\frac{n\pi}{A}\right) a_n, \tag{5.18}$$

$$8n \sum_{m=1}^{\infty} \frac{f_m(0)}{4n^2-(2m-1)^2} = -\frac{2}{n} + \pi I_0\left(\frac{n\pi}{A}\right) a_n. \tag{5.19}$$

From these two equations, we get an infinite set of linear algebraic equations for the $f_m(0)$, which can be solved by a truncation procedure. The constants a_n may easily be obtained once the $f_m(0)$ have been found. Second, the right-hand side may be rearranged as the series expansion of $\cos[(2m-1)\pi(z+A)/2A]$. In this case, we get an infinite set of linear algebraic equations for the a_n. The procedure is similar to that for the first case, and we do not give the explicit formulae.

These two methods were tried and compared. The number of terms to be retained is determined by the criterion that the relative difference between the results of the two methods be sufficiently small in comparison with unity. Thus we found that 60–80 terms are sufficient. We give the inner temperature fields and the streamlines in the Stewartson $E^{\frac{1}{3}}$ layer for the cases $\alpha = 0$, 1, 10 and ∞ for $A = 1$ and 5 in figures 1–4. The case $\alpha = 0$ corresponds to that of an incompressible fluid. As the temperature perturbation Φ_0 is identically zero in this case, it is not shown in the figures. In the opposite limit $\alpha = \infty$, there is no closed circulation in the Stewartson layer, so this is not shown either.

6. Discussion

From the above results we can conclude that (*a*) the flow fields and inner temperature fields in the cases of a conducting and an insulated side wall are the same (to zeroth order in h) if the condition $h \ll E^{\frac{1}{3}}$ holds and we disregard non-linear convective heat transport, and (*b*) the closed circulation in the $E^{\frac{1}{3}}$ layer is strongly suppressed and the inner temperature field deviates from the linear profile if $h \gtrsim E^{\frac{1}{3}}$ and the side wall is thermally insulated.

In a practical gas centrifuge, h is about 0·2 and $E^{\frac{1}{3}} \sim 10^{-2}$, so $h \gg E^{\frac{1}{3}}$ does hold. Therefore the thermal condition on the side wall is important in the determination of all physical variables except the axial velocity profile in the inner core.

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